COLLEGE ALGEBRA STUDENTS' ATTITUDES TOWARD MATH AND GRAPHS: AN EXPLORATORY FACTOR ANALYSIS

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We report on results from a mixed methods study investigating a measure of students' attitudes toward math and graphs. At the beginning of eight consecutive fall and spring semesters, we distributed a fully online attitude survey, adapted from Pepin (2011), to undergraduate College Algebra students. Our report includes two samples, Validation (n=1256) and Calibration (n=712). Our research team qualitatively coded students' responses into five categories: positive, mixed, ambiguous, negative, detached (Gardner et al., 2019). Next, we quantitized those qualitative codes into a four category scale, which condensed the mixed and ambiguous categories. Conducting an Exploratory Factor Analysis, we found that students' attitudes grouped by topics (math and graphs). We conclude with implications for research and practice.

Keywords: Affect, Emotion, Beliefs, and Attitudes, Data Analysis and Statistics, Research Methods, Undergraduate Education

We report on a mixed methods study in which we examine undergraduate students' attitudes toward math and graphs. Our population is students who enroll in College Algebra, one of the earliest credit-bearing mathematics courses at U.S. institutions (Blair et al., 2018), with a history of challenges for student success (Gordon, 2008; Tunstall, 2018). Analyzing students' responses to a fully online, open-ended survey, we investigate a measure of students' attitudes. We aim to contribute to the field's knowledge about the attitudes of College Algebra students.

A key aspect of our study is the quantitizing of qualitative data (Sandelowski et al., 2009). Quantitizing is a process of assigning numerical values to non-numerical data. In so doing, researchers can illuminate patterns and peculiarities arising within large data sets via quantitative analysis strategies. Our quantitizing has allowed us to transform qualitative attitude codes into a four-category scale, which then afforded an exploratory factor analysis.

Theoretical Framework

We adopt Di Martino and Zan's (2010) multidimensional perspective on students' attitudes toward math. That is, students' attitudes toward math comprise three interrelated dimensions: their emotional disposition toward math (e.g., how they like or dislike math), their perceived competence toward math (e.g., how they feel about their capabilities when it comes to math), and their vision of what math is (e.g., what they view math to be). This stance blurs boundaries between McLeod's (1992) categories of beliefs, attitudes, and emotions as distinct components of mathematical affect.

Drawing on this perspective, Pepin and colleagues (Ding et al., 2015; Pepin, 2011) developed a survey including three open-ended response questions, addressing each of these dimensions. In their analysis, they coded students' responses according to three categories: positive, negative, and neutral. Yet, students' attitudes toward math could have complexities beyond just a positive or negative attitude (e.g., Di Martino, 2019). For example, a student could feel that mathematics is "boring and cool at the same time." Hence, future studies could make room for complexities in students' attitudes, via instruments and coding mechanisms.

Methods

The Attitude Survey

We adapted the attitude survey that Pepin and colleagues (Ding et al., 2015; Pepin, 2011) to include five open-ended response questions and to be in a fully online format (Johnson et al., 2019). Table 1 shows the survey questions. The first three questions were the same as those used by Pepin and colleagues. We added the last two questions to address students' attitudes toward graphs in two areas: emotional disposition (like/dislike) and perceived competence (can/cannot). We decided to include students' attitudes toward graphs, because students worked with graphing activities as part of the broader research projects of which this study was part. To allow for a range of responses, we did not require students to select like/dislike or can/cannot. Students were to type in responses to each of the questions; they could complete the survey on a mobile phone, tablet, or computer.

Table 1: The Attitude Survey

Attitude Survey Questions
I like/dislike math because____
I can/cannot do math because_____
Mathematics is______
I like/dislike graphs because _____
I can/cannot make sense of graphs because_____

Data Collection

We collected data in conjunction with two U.S. National Science Foundation funded research projects, examining undergraduate college algebra students and instructors. One aspect of both projects was to investigate students' attitudes toward mathematics. The first project took place at a single institution and concluded in Summer 2020. The second project began in Fall 2020, and extended the efforts to four institutions.

Across eight consecutive spring and fall semesters (Spring 2018 through Fall 2021), we administered the attitude survey to students enrolled in college algebra. In our broader study, we administered the attitude survey in both the beginning and end of the semester. For this analysis, we drew on only those responses from the beginning of the semester. We made this choice to allow for a greater sample size.

We separated our data into two samples. The validation sample (n=1,256) was collected from students who responded between Spring 2018 and Spring 2021. The calibration sample (n=712) was collected from students who responded in Fall 2021. The Fall 2021 sample was greater than other semesters because student responses included a large, lecture style course at one of the institutions. Hence, we decided to use responses from that semester to calibrate our findings with the validation sample.

Data Analysis

We mixed qualitative and quantitative methods for data analysis. First, we engaged in qualitative coding, following the coding scheme put forward by Gardner et al. (2019). Second, we quantitized the qualitative data (Sandelowski et al., 2009), to turn qualitative codes into a scale via Rasch Analysis (Bond et al., 2015). Third, we conducted an Exploratory Factor Analysis (EFA) to examine construct validity.

Qualitative Coding. We drew on students' responses to the five questions in Table 1 as sources of data for their attitude towards mathematics. We coded their responses to extend beyond binary choices of positive or negative (Gardner et al., 2019). Table 2 lists the five categories, including a brief description and sample response. Along with positive and negative, we added the codes of Mixed and Ambiguous, to indicate when student responses evidenced more than one code (Mixed) or when student responses could be coded as positive or negative (Ambiguous). In addition, we included the code of Detached, to indicate when a student separated the content of math or graphs from their connection to it. As cautioned by Di Martino (2019), students' statements about the utility of mathematics (e.g., "math is useful" which we coded as detached) pointed to something other than a positive attitude toward mathematics.

Table 2: Attitude Survey Qualitative Codes

Code	Description	Sample Response
Positive	Like/Can	I like graphs because they represent what a function will look like
Mixed	More than one of these codes: positive, ambiguous, and/or negative	Mathematics is boring and cool at the same time.
Ambiguous	Code could be positive or negative	It is hard for me.
Negative	Dislike/Cannot	I cannot do mathematics because I forget the steps
Detached	Separates the content from their connection to it.	Mathematics is filled with a lot of rules!

For the validation sample, each question was coded by two trained coders. Each coder received training with a coding rubric, then independently coded responses. After coding, they met to identify disagreements, and then calibrated the disagreements via discussion, consulting with an expert coder if needed. As the data set grew, our team trained a machine to assist with the qualitative coding process. Beginning in Spring 2021, we used the machine learning program to assist with the qualitative coding process, verifying with human coders via the earlier process if there was less than 70% confidence by the machine coding.

Quantitizing the qualitative codes: Rasch analysis. Via Rasch analysis (Bond et al., 2015), we transformed the qualitative codes into a mathematically supported scale, ordering the codes according to the level of positivity expressed by each category. This resulted in a collapsing of the Mixed and Ambiguous codes. Our scale was as follows: 0-Detached, 1-Negative, 2-Mixed/Ambiguous, 3-Positive. Category probability curves indicated an even distribution of the four categories with clearly advancing steps. Rasch-Andrich thresholds increased with category values with no evidence of step misfit. Per Linacre (2015), Mean Square (MNSQ) infit values should be less than 2.0; our values ranged from 0.91 to 1.24.

Exploratory factor analysis (EFA). We conducted EFA to examine construct validity. Specifically, to explore how items grouped together mathematically compared to the intended theoretical groupings. We used Bartlett's test of sphericity and the Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy to assess the suitability of the items for factor analysis. We assessed dimensionality using principal components analysis.

Results

Our analysis revealed that the samples were adequate and the items were suitable for factor analysis. The Validation sample (KMO = 0.549) and the Calibration sample (KMO= 0.524) were over the 0.5 minimum threshold. Bartlett's test was significant at $p < .001$ with a χ^2= 548.89 validation sample, and a χ^2= 282.17 for calibration sample.

For the validation sample, two factors were retained, the first with eigenvalues of 1.698 and 1.192, combined explaining 57.8% of the total variance (Table 3). Two factors were likewise retained for the Calibration sample (eigenvalues of 1.60 and 1.24), explaining 56.71% of the total variance. Using a Varimax rotation, the first factor we called Attitude Towards Math; it included the first three questions from Pepin (2011, Table 1). The second factor we called Attitude Towards Graphs; it included the two new questions about graphs.

Table 3: Attitude Survey Item Statistics

Item	Calibration Mean	Calibration Factor Loading	Validation Mean	Validation Factor Loading
I like/dislike math because____	2.09	0.68	2.07	0.66
I can/cannot do math because_____	2.39	0.61	2.35	0.61
Mathematics is______	1.81	0.13	1.85	0.21
I like/dislike graphs because _____	2.10	0.71	2.12	0.70
I can/cannot make sense of graphs because_____	2.30	0.71	2.28	0.69

Discussion

The results of our EFA analysis reveal that items loaded by topic: math and graphs. Hence, students' emotional disposition or perceived competence toward math and graphs may not align with each other. Our analysis points to interrelationships between dimensions of attitudes put forward by Di Martino and Zan (2010).

We found a difference between the attitudes coded in the Validation (n=1256) and Calibration (n=712) samples. Interesting, the Validation sample had more negative codes while the Calibration sample had more positive codes. In the Validation sample, 28% of responses were coded positive and 41% negative. In the Calibration sample 46% of responses were coded as positive and 23% coded negative. We conjectured that the Calibration sample had more positive attitudes in part because students were returning to in-person learning that semester.

We address two limitations. Because our data sources were limited to students' responses to the attitude surveys at the beginning of the semester, our results report only students' attitudes at the start of the course. Furthermore, our resulting attitude scale condenses Mixed and Ambiguous to a single code, potentially dampening some complexities in students' attitudes.

Our study contributes to research investigating attitudes toward math and graphs in an under-studied population, undergraduate students in early credit bearing mathematics courses, such as College algebra. Students' mean response codes (Table 3) indicate that they entered College Algebra with more positive than negative emotional dispositions and perceived competence toward both math and graphs. Future studies can investigate links between students' attitudes toward math and graphs and their engagement in the course.

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